

國立臺中教育大學 109 學年度學士班日間部轉學生招生考試

微積分試題

【本考科得以鉛筆作答】

適用學系：數學教育學系二、三年級

一、填充題（每題 4%，共 84%）

1. Evaluate $\lim_{x \rightarrow 0} \frac{|3x-2|-|3x+2|}{x}$.

2. Let $f(x) = \begin{cases} \frac{1-\cos x}{\sin x} & \text{if } x > 0 \\ mx+b & \text{if } x \leq 0 \end{cases}$. Suppose that f is differentiable on $(-\pi, \pi)$. Find the constants m and b .

3. Let $f(x) = (\cos x)^x$. Find $f'(x)$.

4. At what points does the curve $x^3 + y^3 = 6xy$ have a horizontal tangent?

5. A box with a square base and open top must have a volume of $32,000 \text{ cm}^3$. Find the dimensions of the box that minimize the amount of material used.

6. Find the linearization of $f(x) = 2 - \int_2^{x+1} \frac{9}{1+t} dt$ at $x = 1$.

7. Suppose that f is continuous, $f(0) = 0$, $f(2) = 1$, $f'(x) > 0 \quad \forall x \in \mathbb{R}$, and $\int_0^2 f(x) dx = \frac{1}{3}$. Find the value of the integral $\int_0^1 f^{-1}(y) dy$.

8. Let $4x^2 - 2y^2 = 9$. Then $\frac{d^2y}{dx^2} = \underline{\hspace{2cm}}$.

9. Calculate $\int_0^2 x(x^2+1)^3 dx = \underline{\hspace{2cm}}$.

10. Let $f(x) = x^5 + x + 1$. Then $(f^{-1})'(1) = \underline{\hspace{2cm}}$.

11. Evaluate $\int_0^{\ln 3} e^x \sqrt{1 + e^x} dx = \underline{\hspace{2cm}}$.

12. Find $\lim_{x \rightarrow 0} (1 + \sin x)^{1/x} = \underline{\hspace{2cm}}$.

13. Evaluate $\int_1^{\sqrt{2}} \frac{dx}{x^2 \sqrt{4-x^2}} = \underline{\hspace{2cm}}$.

14. Evaluate $\int_1^{+\infty} \frac{dx}{x^3} = \underline{\hspace{2cm}}$.

15. 求 $\lim_{x \rightarrow 0} \frac{2 \tan x}{x} = \underline{\hspace{2cm}}$.

16. 求 $\lim_{x \rightarrow 0} \frac{e^x}{x} = \underline{\hspace{2cm}}$.

17. 請問級數 $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ ，是否收斂？ $\underline{\hspace{2cm}}$.

18. 請問級數 $\sum_{n=1}^{\infty} \frac{e^{2n}}{n^n}$ ，是否收斂？ $\underline{\hspace{2cm}}$.

19. 請求出一個以 0 為中心的冪級數(power series)代表 $g(x) = \tan^{-1}x$ ？ $\underline{\hspace{2cm}}$ (請至少寫出前三項)，而其收斂區間為何？ $\underline{\hspace{2cm}}$.

20. 求 $\int_{-\infty}^0 x e^x dx = \underline{\hspace{2cm}}$.

21. 求 $\int_{-1}^1 \frac{1}{x^2} dx = \underline{\hspace{2cm}}$.

二、計算證明題（每題 8%，共 16%）

1. Let $f(x, y) = \begin{cases} \frac{xy^2}{x^2 + y^4} & \text{if } (x, y) \neq (0, 0), \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$

(a) Show that f is not continuous at $(0, 0)$.

(b) Find $f_x(x, y)$.

2. Evaluate the integral $\int_0^1 \int_0^1 e^{\max\{x^2, y^2\}} dy dx$, where $\max\{x^2, y^2\}$ means the larger of the numbers x^2 and y^2 .